The Magnetic Forces القوى المغناطيسية

8.1 Introduction

The electric field causes a force to be exerted on a charge that may be either **stationary** or in **motion**; we will see that the steady magnetic field is capable of exerting a force only on a ***moving*** charge. This result appears reasonable; a magnetic field may be produced by moving charges and may exert forces on moving charges; a ***magnetic field cannot arise from stationary charges and cannot exert any force on a stationary charge***

8.2 Force on a Moving Charge القوة على شحنة متحركة

In an electric field, the definition of the electric field intensity shows us that the force on a charged particle is

$$F = QE$$

The force is in the same direction as the electric field intensity (for a positive charge) and is directly proportional to both **E** and *Q*. If the charge is in motion, the force at any point in its trajectory is then given by equation above.

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A charged particle in motion in a magnetic field of flux density B is found experimentally to experience a force whose magnitude is ***proportional to the product of the magnitudes of the charge Q, its velocity v, and the flux density B, and to the sine of the angle between the vectors v and B***. The direction of the force is perpendicular to both **v** and **B** and is given by a unit vector in the direction of **v**×**B**. The force may therefore be expressed as

$$F = Qv \times B$$

Therefore, the direction of a particle in motion can be changed by a magnetic field. The magnitude of the velocity, v, and consequently the kinetic energy, will remain the same. This is in contrast to an electric field, where the force F=QE does work on the particle and therefore changes its kinetic energy.

The **force** on a **moving particle** arising from combined **electric** and **magnetic** fields is obtained easily by superposition

$$F = Q(E + v \times B)$$

This equation is known as the ***Lorentz force*** *equation*

Example: The point charge **Q = 18n**C has a velocity of 5×10^6 m/s in the direction $a_v = 0.60\mathbf{a_x} + 0.75\mathbf{a_y} + 0.3\mathbf{a_z}$. Calculate the magnitude of the force exerted on the charge by the field: (a) $B = -3\mathbf{a_x} + 4\mathbf{a_y} + 6\mathbf{a_z}$ mT; (b) $E = -3\mathbf{a_x} + 4\mathbf{a_y} + 6\mathbf{a_z}$ kV/m; (c) B and E acting together?

Solution:

(a)

$$F_m = Qv \times B = 18 * 10^{-9} * 5 * 10^6 \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ 0.6 & 0.75 & 0.3 \\ -3 & 4 & 6 \end{vmatrix} * 10^{-3}$$

$$= 9 * 10^5 (3.3\mathrm{a}_x - 4.5\mathrm{a}_y + 4.65\mathrm{a}_z) = 653.7\ \mu N$$

(b)

$$F_e = QE = 18 * 10^{-9} (-3\mathrm{a}_x + 4\mathrm{a}_y + 6\mathrm{a}_z) * 10^3 = 140.5\ \mu N$$

(c)

$$F = Q(E + v \times B) = 18 * 10^{-9} [-3\mathrm{a}_x + 4\mathrm{a}_y + 6\mathrm{a}_z + 5(3.3\mathrm{a}_x - 4.5\mathrm{a}_y + 4.65\mathrm{a}_z)]$$

$$F = 18 * 10^{-9} (13.5\mathrm{a}_x - 18.5\mathrm{a}_y + 29.25\mathrm{a}_z) = 668.6\ \mu N$$

8.3 Magnetic Force on a Current Element

A frequently encountered situation is that of a current-carrying conductor in an external magnetic field. Since $= \frac{dQ}{dt}$, the differential force equation may be written

$$dF = dQ(v \times B) = Idt(v \times B) = I(dL \times B)$$

$$F = I \oint B \times dL$$

where dL = v dt is the elementary length in the direction of the conventional current

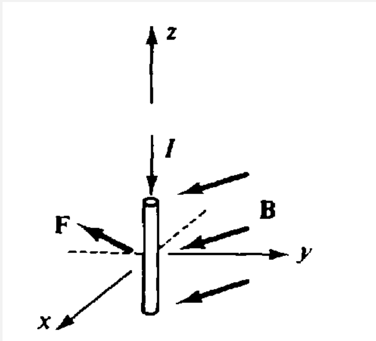
If the conductor is straight and the field is constant along it, the differential force may be integrated to give

$$F = IL \times B = ILB \sin\theta$$

where θ is the angle between the vectors representing the direction of the current flow and the direction of the magnetic flux density

Example: Find the **force** on a **straight conductor** of length 0.30 m carrying a current of 5.0 A in the $-\mathbf{a_z}$, direction, where the field is B = $3.5 \times 10^{-3}(\mathbf{a_x} - \mathbf{a_y})$ T?

Solution:



$$F = IL \times B$$

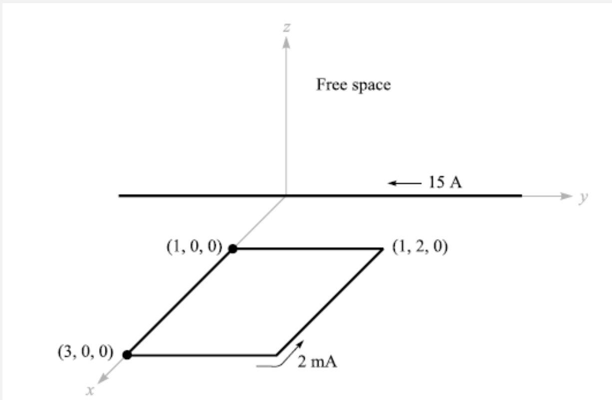
$$F = 5\left[-0.3\mathrm{a}_z \times 3.5 * 10^{-3}\left(\mathrm{a}_x - \mathrm{a}_y\right)\right]$$

$$F = 5.25\left(-\mathrm{a}_x - \mathrm{a}_y\right) mN$$

$$F = 7.42\ mN$$

Example: a square loop of wire in the z = 0 plane carrying 2 mA in the field of an infinite filament on the y axis, as shown. Find the total force on the loop?

Solution:



$$H = \frac{I}{2\pi}\left(\frac{-z\mathrm{a}_x + x\mathrm{a}_z}{z^2 + y^2}\right)$$

$$H = \frac{I}{2\pi x}\mathrm{a}_z = \frac{15}{2\pi x}\mathrm{a}_z$$

$$B = \mu H = 4\pi \times 10^{-7}\frac{15}{2\pi x}\mathrm{a}_z$$

$$B = \frac{3 * 10^{-6}}{x}\mathrm{a}_z$$

$$F = I\oint B \times dL$$

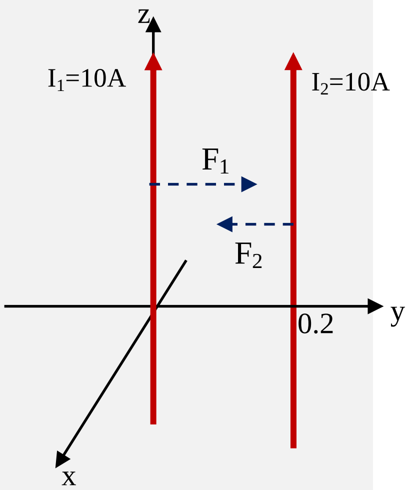
$$F = -6 * 10^{-9}\left[\int_{x=1}^{3}\frac{1}{x}\mathrm{a}_z \times dx\mathrm{a}_x + \int_{y=0}^{2}\frac{1}{x}\mathrm{a}_z \times dy\mathrm{a}_y + \int_{y=0}^{2}\frac{1}{x}\mathrm{a}_z \times dx\mathrm{a}_x + \int_{y=2}^{0}\frac{1}{x}\mathrm{a}_z \times dy\mathrm{a}_y\right]$$

$$F = -6 * 10^{-9}\left[[\ln x]_1^3\mathrm{a}_y - \frac{1}{3}[y]_0^2\,\mathrm{a}_x + [\ln x]_3^1\mathrm{a}_y - [y]_2^0\,\mathrm{a}_x\right]$$

$$F = -6 * 10^{-9}\left[\frac{-2}{3}\mathrm{a}_x + 2\mathrm{a}_x\right] = -8\mathrm{a}_x\ nN$$

Example: Find the forces per unit length on two long, straight, parallel conductors if each carries a current of 10 A in the same direction and the separation distance is 0.2 m?

Solution:



$$H_1 = \frac{I_1}{2\pi}\left(\frac{-y\mathrm{a}_x + x\mathrm{a}_y}{y^2 + x^2}\right)$$

$$H_1 = \frac{-I_1}{2\pi y}\mathrm{a}_x = \frac{-10}{2\pi(0.2)}\mathrm{a}_x$$

$$H_1 = \frac{-25}{\pi}\mathrm{a}_x$$

$$B_1 = \mu H_1 = 4\pi \times 10^{-7}\frac{-25}{\pi}\mathrm{a}_x$$

$$B_1 = -10^{-5}\mathrm{a}_x$$

$$F_2 = I_2 L_2 \times B_1$$

$$= 10(L\mathrm{a}_z \times -10^{-5}\mathrm{a}_x)$$

$$\frac{F_2}{L} = -10^{-4}\mathrm{a}_y$$

$$H_2 = \frac{I_2}{2\pi}\left(\frac{-(y-y_0)\mathrm{a}_x + (x-x_0)\mathrm{a}_y}{(y-y_0)^2 + (x-x_0)^2}\right)$$

$$H_2 = \frac{-I_2}{2\pi(y-y_0)}\mathrm{a}_x = \frac{-10}{2\pi(0-0.2)}\mathrm{a}_x$$

$$H_2 = \frac{25}{\pi}\mathrm{a}_x$$

$$B_2 = \mu H_2 = 4\pi \times 10^{-7}\frac{25}{\pi}\mathrm{a}_x$$

$$B_2 = 10^{-5}\mathrm{a}_x$$

$$F_1 = I_1 L_1 \times B_2$$

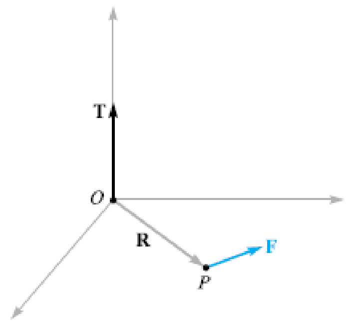
$$= 10(L\mathrm{a}_z \times 10^{-5}\mathrm{a}_x)$$

$$\frac{F_2}{L} = 10^{-4}\mathrm{a}_y$$

8.4 Torque

The moment of a force or torque about a specified point is the cross product of the lever arm about that point and the force. The lever arm, r, is directed from the point about which the torque is to be obtained to the point of application of the force. In Fig. 10-5 the force at P has a torque about O given by

$$T = \mathrm{r} \times F$$



Example: A conductor located at x = 0.4m, y =0 and 0< z <2.0 m carries a current of 5.0 A in the $\mathbf{a_z}$ direction. Along the length of the conductor B = 2.5$\mathbf{a_x}$ T. Find the torque about the z axis?

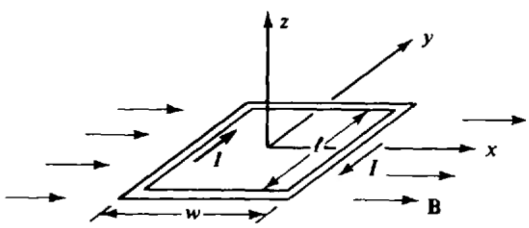
Solution:

$$F = IL \times B = 5(2\mathrm{a}_z \times 2.5\mathrm{a}_x) = 25\mathrm{a}_y \ \ N$$

$$T = \mathrm{r} \times F = 0.4\mathrm{a}_x \times 25\mathrm{a}_y = 10\mathrm{a}_z \ \ N.m$$

8.5 Magnetic Moment of a Planar Coil

Consider the single-turn coil in the z = 0 plane shown in Fig. below, of width w in the x direction and length *l* along y. The field B is uniform and in the +x direction. Only the ±y directed currents give rise to forces. For the side on the left,



$$\boldsymbol{F} = \boldsymbol{I}(\boldsymbol{L}\mathbf{a}_{\boldsymbol{y}} \times \boldsymbol{B}\mathbf{a}_{\boldsymbol{x}}) = -\boldsymbol{BIL}\mathbf{a}_{\boldsymbol{z}}$$

and for the side on the right,

$$\boldsymbol{F} = \boldsymbol{BIL}\mathbf{a}_{\boldsymbol{z}}$$

The torque about the y axis from the left current element requires a lever arm r = -(w/2)$\mathbf{a}_x$; the sign will change for the lever arm to the right current element. The torque from both elements is

$$T = \mathrm{r} \times F = \left(\frac{-w}{2}\right)\mathbf{a}_x \times -\boldsymbol{BIL}\mathbf{a}_z + \left(\frac{w}{2}\right)\mathbf{a}_x \times \boldsymbol{BIL}\mathbf{a}_z = \boldsymbol{BILw}(-\mathbf{a}_y)$$

$$T = BIA(-\mathbf{a}_y)$$

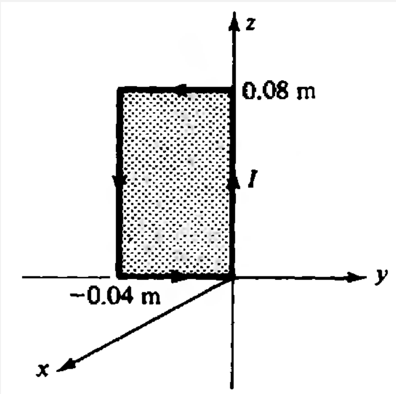
where A is the area of the coil. It can be shown that this expression for the torque holds for a flat coil of arbitrary shape

The ***magnetic moment*** m of a planar current loop is defined as IAa_N , where the unit normal a_n is determined by the right-hand rule. It is seen that the torque on a planar coil is related to the applied field by

$$T = m \times B$$

$$m = IA\,\mathrm{a}_N$$

Example: The rectangular coil in Figure below is in a field $B = 0.05\frac{(\mathrm{a}_x+\mathrm{a}_y)}{\sqrt{2}}\,T$ (a) Find the torque about the z axis when the coil is in the position shown and carries a current of 5.0 A (b)Find the torque if the coil turns through 45°?



Solution:

(a)

$$m = IA\,\mathrm{a}_N = 5 * 0.08 * 0.04\mathbf{a}_x = 0.016\,\mathbf{a}_x$$

$$T = m \times B = 0.016\,\mathbf{a}_x \times 0.05\frac{(\mathrm{a}_x + \mathrm{a}_y)}{\sqrt{2}} = 5.66 * 10^{-4}\,\mathbf{a}_z$$

(b)

If the coil turns through 45^0 the direction of m will be:$\frac{a_x+a_y}{\sqrt{2}}$

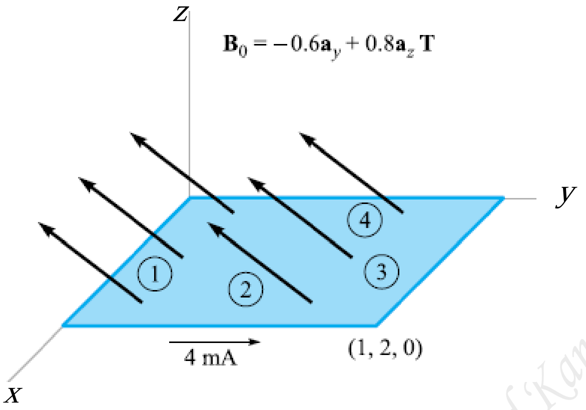
$$T = m \times B = 0.016\,\frac{a_x + a_y}{\sqrt{2}} \times 0.05\frac{(\mathrm{a}_x + \mathrm{a}_y)}{\sqrt{2}} = 0$$

Homework

Q₁: *A point charge for which Q = 2 × 10⁻¹⁶ C is moving in the combined fields* E = 100$\mathbf{a}_x$ - 200$\mathbf{a}_y$ + 300$\mathbf{a}_z$ *V/m and* B = -3$\mathbf{a}_x$ + 2$\mathbf{a}_y$- $\mathbf{a}_z$ *mT. If the charge velocity* v = (2$\mathbf{a}_x$ -3$\mathbf{a}_y$ - 4$\mathbf{a}_z$) 10^5 *m/s, find the unit vector showing the direction in which the charge is accelerating?*

Ans:.0.7a_x + 0.7a_y – 0.12a_z

Q₂: *Consider the rectangular loop shown in Figure below. Calculate the torque?*



Ans: *4.8*a_x *mN*.m

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Q₃: *A conducting filamentary triangle joins points A(3, 1, 1), B(5, 4, 2), and C(1, 2, 4). The segment AB carries a current of 0.2 A in the* $\mathbf{a}_{AB}$ *direction. There is present a magnetic field* $\mathbf{B}$ *= 0.2*$\mathbf{a}_x$ *−0.1*$\mathbf{a}_y$ *+ 0.3*$\mathbf{a}_z$ *T. Find: (a) the force on segment BC; (b) the force on the triangular loop; (c) the torque on the loop about an origin at A; (d) the torque on the loop about an origin at C.*

Ans: -0.08$\mathbf{a}_x$ +0.32$\mathbf{a}_y$ +0.16$\mathbf{a}_z$ N; **0**; -0.16$\mathbf{a}_x$ -0.08$\mathbf{a}_y$ +0.08$\mathbf{a}_z$ N. m; -0.16$\mathbf{a}_x$ -0.08$\mathbf{a}_y$ + 0.08$\mathbf{a}_z$ N.m

Q₄: *A circular current loop of radius ρ and current I lies in the z = 0 plane. Find the Torque which results if the current is in the* a_ϕ *direction and there is a uniform field* $B = B_o\,(a_x + a_y)/\sqrt{2}$

Ans: $\pi\rho^2 B_o I/\sqrt{2}\ a_y$